

# Readers' Forum

Brief discussions of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one Journal page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.

AIAA 81-4332

## Comment on "Propagation of Weak MHD Waves in Steady Hypersonic Flows with Radiation"

Vishnu D. Sharma\*

University of Maryland, College Park, Md.

### Introduction

A RECENT Note by Ram and Pandey<sup>1</sup> is full of serious mistakes. Although they have reproduced the arguments used by Jeffrey and Taniuti,<sup>2</sup> they seem to have misunderstood these arguments, and therefore they have failed to properly translate them to their problem. Thus, their final result as well as several intermediate steps are incorrect. Unfortunately, the column matrix  $B$  in Eq. (2) of Ref. 1 is wrongly stated. A major correction is required for the eigenvectors  $L^1$ ,  $L^2$ ,  $L^3$ , and  $L^4$  of the matrix  $A$  in their Eq. (2); note that these are *left* eigenvectors, which are therefore row vectors. However, they appear as column vectors with incorrect components, thus leading to wrong results, apart from absurd matrix multiplication; see, for example, their Eqs. (3), (4), (6), (7) and (12), where we come across the matrix product of the form  $LB$ ,  $LU$  and  $L\pi$ , where all  $L$ ,  $B$ ,  $U$  and  $\pi$  are column vectors with five components. In Ref. 1, the constants  $C_1$  and  $C_2$ , which play an important role in deciding the growth and decay behavior of the wave, are incorrectly computed; thus, the discussion in the last section of Ref. 1 needs an essential modification. The purpose of the present Comment is to rectify these serious errors, mainly because they are likely to lead to misconceptions in the minds of readers and future workers, who may need these basic equations and the technique of Jeffrey and Taniuti<sup>2</sup> to discuss various other aspects of the problem or similar problems. In the following section, we derive the correct expression for the wave amplitudes; for the sake of precision and clarity, a few intermediate steps involving simple computations have been deleted.

### Behavior at the Wave Front

In this section, the symbols and notations used are as in Ref. 1.

The column matrix  $B$  in Eq. (2) of Ref. 1 is in error; in fact, it should read as

$$B = \frac{4(\gamma-1)\alpha_p a_R T^4}{q(q^2 - c^2 - b^2)} \begin{bmatrix} (q^2 - b^2) \\ -q/\rho \\ 0 \\ b^2 \\ 1 \end{bmatrix}$$

where  $b = (2h/\rho)^{1/2}$ . It may be noted that  $h = (\mu H^2/2)$ , where  $H$  is the magnetic field strength *transverse* to the trajectories of fluid particles and  $\mu$  is the magnetic permeability.

In Ref. 1,  $L^j$  ( $j=1, 2, 3, 4, 5$ ) are the *left* eigenvectors of the matrix  $A$  corresponding to the eigenvalues  $\lambda^j$ , and therefore they are the *row* vectors, instead of column vectors computed therein. The correct eigenvectors are therefore

$$L^1 = [(\rho q^2 Q)^{-1} \quad 0 \quad 1 \quad (\rho q^2 Q)^{-1} \quad 0]$$

$$L^2 = [(\rho q^2 Q)^{-1} \quad 0 \quad -1 \quad (\rho q^2 Q)^{-1} \quad 0]$$

$$L^3 = [(I + M_f^2)/\rho q \quad 1 \quad 0 \quad 0 \quad 0]$$

$$L^4 = [-M_f^2 \quad 0 \quad 0 \quad 1 \quad 0]$$

$$L^5 = [-M^2/q^2 \quad 0 \quad 0 \quad 0 \quad 1]$$

where  $Q$ ,  $M$  and  $M_f$  are same as in Ref. 1.

In Ref. 1, the transformation, in fact, is made from  $(s, n)$  coordinates to  $(\psi, \phi)$  coordinates with  $\psi = s$  and  $\phi_s + \lambda' \phi_n = 0$ , but they have repeatedly stated incorrectly that the transformation is from  $(s, n)$  to  $(\phi, \psi)$ . Further, the relations stated as Eq. (11) in Ref. 1 are incorrect; in fact, they should read as

$$\pi_1 - \rho_0 q_0^2 Q_0 \pi_3 + \pi_4 = 0$$

$$(I + M_f^2)_0 \pi_1 + \pi_2 = 0$$

$$-M_{f0}^2 \pi_1 + \pi_4 = 0$$

$$-\pi_1 + c_0^2 \pi_5 = 0$$

Equation (12) of Ref. 1 is incorrect, in fact it should read as

$$\begin{aligned} [\nabla_u (L^1 B)]_0 \pi &= \frac{4(\gamma-1)\alpha_p a_R T_0^4}{\rho_0 c_0^2 (I + M_f^2)_0^{3/2} (M^2 - M_f^2 - I)_0^{3/2}} \\ &\times \left\{ \frac{M_0^2 (2M_f^2 + \gamma + I)_0 - 2(2-\gamma)}{2(I + M_f^2)_0} + \frac{(I + M_f^2)_0^2}{M_0^2} \right. \\ &\times (M^2 - M_f^2 - I)_0 (\gamma p_0 \frac{\partial}{\partial p} (\ln \alpha_p) + 4\gamma - 1/2) - 4M_{f0}^2 \left. \right\} \pi_2 \end{aligned}$$

Thus, in Ref. 1, the modified form of Eq. (13) is

$$\pi_{2\psi} + C_1 \pi_2 = 0$$

where

$$C_1 = -1/2 q_0 (I + M_f^2)_0 Q_0 [\nabla_u (L^1 B)]_0 \pi$$

it may be noted that the sign of  $C_1$  will depend on the sign of the quantity  $[\nabla_u (L^1 B)]_0 \pi$ , computed above.

Further, Eq. (14) of Ref. 1 is incorrect; in fact, it should read as

$$\pi = \bar{\pi}_2 \exp(-C_1 \psi) \begin{bmatrix} -\rho_0 q_0 / (1 + M_{f0}^2)_0 \\ I \\ -I / (q_0 Q_0) \\ -\rho_0 q_0 M_{f0}^2 / (1 + M_{f0}^2)_0 \\ -\rho_0 q_0 / (c_0^2 (1 + M_{f0}^2)_0) \end{bmatrix}$$

where  $\bar{\pi}_2 = \lim_{\psi \rightarrow 0} \pi_2$ .

Further, Eq. (15) of Ref. 1 is incorrect; it should read as  $n_{\phi\psi} = -C_2 \pi_2$ , where

$$C_2 = [(2 - \gamma) q_0 M_{f0}^2 M_0^2 + c_0 M_0 (1 + M_{f0}^2)_0^{3/2} (2(1 + M_{f0}^2)_0 + M_0^2 (\gamma - 1))] / [2c_0^2 (1 + M_{f0}^2)_0^{3/2} (M^2 - M_{f0}^2 - 1)_0^{3/2}]$$

Since for real gases  $1 < \gamma < 2$ , it follows that  $C_2$  is positive.

Thus the behavior of the wave amplitude given by Eq. (19) of Ref. 1 depends critically on the sign of  $C_1$ . For instance:

1) When  $\text{sgn}(0) = -\text{sgn} C_2$ , then it follows that for  $C_1 > 0$ ,  $\lim_{s \rightarrow \infty} a(s) = 0$  (i.e., the wave damps out); and for  $C_1 < 0$ ,  $\lim_{s \rightarrow \infty} a(s) = C_1 / C_2$ , i.e., the wave ultimately takes a stable wave form.

2) When  $\text{sgn}(0) = \text{sgn} C_2$ , then i) for  $C_1 > 0$ , there exists a positive critical amplitude  $a_c$  given by  $a_c = C_1 / C_2$ , such that for  $a(0) < a_c$ ,  $a \rightarrow 0$  as  $s \rightarrow \infty$ ; for  $a(0) > a_c$ ,  $a \rightarrow \infty$  as  $s \rightarrow s_c$  (i.e., the wave culminates into a shock wave), where  $s_c$  is given by

$$s_c = \frac{1}{C_1} \ln \left( 1 - \frac{a_c}{a(0)} \right)^{-1}$$

and for  $a(0) = a_c$ ,  $a = a_c$ ; ii) for  $C_1 < 0$ , all waves, no matter how small be their initial amplitude, grow into shock waves at  $s = s_c$  (i.e.,  $a(s) \rightarrow \infty$  as  $s \rightarrow s_c$ ), where  $s_c$  is as given by

$$s_c = (1 / |C_1|) (\ln I + (|C_1| / C_2) a(0))$$

## References

- <sup>1</sup>Ram, R. and Pandey, B. D., "Propagation of Weak MHD Waves in Steady Hypersonic Flows with Radiation," *AIAA Journal*, Vol. 18, July 1980, pp. 855-857.
- <sup>2</sup>Jeffrey, A. and Taniuti, T., *Non-Linear Wave Propagation*, Academic Press, New York, 1964.

AIAA 81-4333

## Comment on "Karman Vortex Shedding and the Effect on Body Motion"

P. K. Stansby\*

University of Manchester, Manchester, England

ERICSSON<sup>1</sup> refers to the writer's experimental investigation of the locking-on of vortex shedding due to the cross-stream vibration of a circular cylinder.<sup>2</sup> Locking-on

at submultiples of the cylinder frequency was reported and a physical explanation was given. However, Ericsson states that the locking-on at half the cylinder frequency (secondary locking-on), observed by the writer, was, in effect, an illusion. This is asserted because secondary locking-on was observed with an amplitude of vibration of 10% of a diameter but not with 29% of a diameter and locking-on had previously been shown to occur more readily with larger amplitudes. The term "locking-on" implies that cylinder motion controls the wake frequency and this definitely occurred (at the lower amplitude); the spectrum of the hot-wire signal outside the wake showed a narrow band at half the cylinder frequency and a marked increase in variance over a small range of cylinder frequencies. (Without locking-on the spectrum at the vortex shedding frequency shows a much wider band.) The mean base suction also showed a small but distinct increase.<sup>3</sup> That secondary locking-on was not observed at the larger amplitude was almost certainly because the frequency range for "stronger" tertiary locking-on (at one third of the cylinder frequency) had increased sufficiently to envelope the range for "weak" secondary locking-on.

## References

- <sup>1</sup>Ericsson, L. E., "Karman Vortex Shedding and the Effect on Body Motion," *AIAA Journal*, Vol. 18, Aug. 1980, pp. 935-944.
- <sup>2</sup>Stansby, P. K., "The Locking-On of Vortex Shedding Due to the Cross-Stream Vibration of Circular Cylinders in Uniform and Shear Flows," *Journal of Fluid Mechanics*, Vol. 74, April 1976, pp. 641-666.
- <sup>3</sup>Stansby, P. K., "Base Pressure of Oscillating Circular Cylinders," *Journal of Engineering Mechanics Division, ASCE*, Vol. 102, Aug. 1976, pp. 591-600.

AIAA 81-4334

## Errata: Solution of the Three-Dimensional Navier-Stokes Equations on a Vector Processor

Lawrence W. Spradley,\* John F. Stalnaker,†  
and Alan W. Ratliff‡

Lockheed-Huntsville Research and Engineering Center,  
Huntsville, Ala.

[AIAA J, 19, 1302-1308 (1981)]

TWO page numbers in this article have inadvertently been switched. Page 1305 should be page 1306, and page 1306 is page 1305.

Received Oct. 21, 1981.

\*Staff Engineer. Member AIAA.

†Scientist, Associate Research. Member AIAA.

‡Group Engineer.

Received Feb. 17, 1981. Copyright © American Institute of Aeronautics and Astronautics, Inc., 1981. All rights reserved.

\*Simon Engineering Laboratories.